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Nonlinear propagation of dust-acoustic waves in a magnetized dusty plasma

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Abstract : Nonlinear propagation of dust-acoustic waves in a cold magnetized dusty plasma, which consists of extremely massive, arbitrarily (negatively or positively) charged inertial dust grains, Boltzmann distributed ions and electrons, has been investigated The reductive perturbation method has been used to derive the Korteweg-de Varies (K-dV) equation which admits a solitonic solution The effects of external magnetic field, obliqueness and polarity of the dust grains on the nature of the dust-acoustic solitons are discussed

Keywords : Magnetized dusty plasma, dust-acoustic solitary waves, Korteweg-de Vries (K-dV) equation

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Nowadays, a considerable attention has been given in understanding different types of collective processes in dusty plasmas which are common in asteroid zones, planetary rings, cometary tails, as well as the lower ionosphere of the earth. It has been found both theoretically [1] and experimentally [2] that the presence of static charged dust grains modifies the existing plasma wave spectra, whereas the dust charge dynamics introduces new eigen modes. The low frequency dust-acoustic mode [1, 2], where the dust particle mass provides the inertia and the pressures of inertialess ions or electrons or both (depending on the polarity of the dust grains) give rise to the restoring force, is one of them. Recently, motivated by these theoretical [1] and experimental [2] studies, Mamun *et al* [3, 4] have reported the nonlinear properties of dust-acoustic waves in a two-component dusty plasma consisting of a negatively charged dust fluid and maxwellian [3] or non-Maxwellian [4] distributed ions. As the effects of obliqueness, external magnetic field, and polarity of the dust grains, which have not been considered in the earlier investigations [1, 3, 4], drastically modify the properties of the electrostatic waves. We, in our present work, have studied the nonlinear propagation of dust-acoustic waves in a cold magnetized three-component dusty plasma which consists of an arbitrarily charged (negatively or positively charged depending on the charging process) dust fluid and Boltzmann distributed ions and electrons.

We consider a three-component dusty plasma which consists of extremely massive, micron-sized arbitrarily (negatively or positively) charged inertial dust grains, and Boltzmann distributed ions and electrons, in the presence of an external static magnetic field $\boldsymbol{B}_0 \parallel \hat{z}$. Thus, at equilibrium we have

$$n_{io} = \nu Z_d n_{do} + n_{eo}, \tag{1}$$

where n_{io}, n_{do}, and n_{eo} are the unperturbed ion, dust, electron number densities, respectively, Z_d is the number of electrons or protons residing on the dust grains, and ν is a parameter which is 1 for electrons – 1 for protons. The dynamics of low phase velocity (lying the ions and dust thermal velocities, *viz*, $v_{td} << v_p << v_{ti}$) dust-acoustic oscillations is governed by [3, 4]

$$\frac{\partial n_d}{\partial t} + \underline{\nabla} \cdot (n_d \underline{u}_d) = 0, \tag{2}$$

$$\frac{\partial \underline{u}_d}{\partial t} + (\underline{u}_d \cdot \underline{\nabla})\, \underline{u}_d = \nu[\underline{\nabla}\phi + \omega_{cd}\, (\hat{z} \times \underline{u}_d)], \tag{3}$$

$$\nabla^2 \phi = \nu\, [\, n_d + \mu_0 e^{\sigma\phi} - \mu_1 e^{-\phi}\,], \tag{4}$$

where n_d is the dust particle number density normalized to n_{do}, $\underline{u}_d$ is the dust fluid velocity normalized to the dust-acoustic speed $C_d = (Z_d T_i / m_d)^{1/2}$ in which T_i being the ion-temperature (in energy units) and m_d being the mass of the charged dust particles, ϕ is the electrostatic wave potential normalized to T_i / e in which e being the magnitude of the electronic charge. $\sigma = T_i / T_e$ with T_e being the electron-temperature (in energy units), $\mu_0 = \mu / (1 - \mu)$ and $\mu_1 = 1 / (1 - \mu)$ with $\mu = n_{eo} / n_{io}$. The time and space variables are in the units of the dust plasma period $\omega_{pd}^{-1} = (m_d / 4\pi\, n_{do} Z_d^2 e^2)^{1/2}$ and the Debye length $\lambda_{Dd} = (T_i / 4\,\pi\, Z_d n_{do} e^2)^{1/2}$, respectively. $\omega_{cd} = (Z_d e B_o / m_d) / \omega_{pd}$ is the dust-cyclotron frequency normalized to ω_{pd}.

To study the propagation of dust-acoustic waves in our dusty plasma model, we construct a weakly nonlinear theory of the dust-acoustic waves with small but finite amplitude which leads to scaling of the independent variables through the stretched coordinates [5, 6]

$$\xi = \varepsilon^{1/2}\, (l_x x + l_y y + l_z z - \upsilon_0 t),$$

$$\tau = \varepsilon^{3/2} t, \tag{5}$$

where ε is a small parameter measuring the weakness of the dispersion. υ_0 is the wave phase velocity normalized to C_d; l_x, l_y, and l_z are the directional cosines of the wave vector $\boldsymbol{k}$ along the X, Y, and Z axes, respectively, so that $l_x^2 + l_y^2 + l_z^2 = 1$. We can expand the perturbed quantities n_d, $u_{dx,y,z}$, and ϕ about their equilibrium values in power of ε, including the terms of $\varepsilon^{3/2}$ [6]

$$\begin{aligned} n_d &= 1 + \varepsilon n_d^{(1)} + \varepsilon^2 n_d^{(2)} + \ldots\ldots\ldots, \\ u_{dx,y} &= 0 + \varepsilon^{3/2} u_{dx,y}^{(1)} + \varepsilon^2 u_{dx,y}^{(2)} + \ldots\ldots\ldots, \\ u_{dz} &= 0 + \varepsilon u_{dz}^{(1)} + \varepsilon^2 u_{dz}^{(2)} + \ldots\ldots\ldots, \\ \phi &= 0 + \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \ldots\ldots\ldots\ldots \end{aligned} \tag{6}$$

Now using (5) and (6) in (2)–(4) one can obtain the first order continuity equation, Z component of momentum equation and Poissons's equation which, after simplification, yield $n_d^{(1)} = \frac{l_z}{\upsilon_0} u_{dz}^{(1)} = -\nu \frac{l_z^2}{\upsilon_0^2} \phi^{(1)}$ and $\upsilon_0 = l_z / \sqrt{\nu [\mu_i + \sigma \mu_o]}$. We can write the first order X and Y components of the momentum equation as $u_{dy}^{(1)} = \frac{l_x}{\omega_{cd}} \frac{\partial\phi^{(1)}}{\partial\xi}$ and $u_{dx}^{(1)} = -\frac{l_y}{\omega_{cd}} \frac{\partial\phi^{(1)}}{\partial\xi}$. These, respectively, represent the Y and X components of the electric field drift. These equations are also satisfied by the second order continuity equation.

Again, using (5) and (6) in (3) and (4), and eliminating $u_{dx,y}^{(1)}$, we obtain the next higher order X and Y components of the momentum equation and Poisson's equation as

$$u_{dx}^{(2)} = -\frac{l_x \upsilon_o}{\nu \omega_{cd}^2} \frac{\partial^2 \phi^{(1)}}{\partial \xi^2}, \tag{7}$$

$$u_{dy}^{(2)} = -\frac{l_y \upsilon_o}{\nu \omega_{cd}^2} \frac{\partial^2 \phi^{(1)}}{\partial \xi^2}, \tag{8}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = \nu n_d^{(2)} + \frac{\nu l_z^2}{\upsilon_o^2} \phi^{(2)} - \frac{\nu}{2} \mu_1 [\phi^{(1)}]^2. \tag{9}$$

The first two of these equations, respectively denotes the Y and X components of the dust polarization drift. Similarly, following the same procedure one can obtain the next higher order continuity equation and Z component of the momentum equation as

$$\frac{\partial n_d^{(1)}}{\partial \tau} - \upsilon_o \frac{\partial n_d^{(2)}}{\partial \xi} + l_x \frac{\partial u_{dx}^{(2)}}{\partial \xi} + l_y \frac{\partial u_{dy}^{(2)}}{\partial \xi} + l_z \frac{\partial}{\partial \xi}\left[u_{dz}^{(2)} + n_{dz}^{(1)} u_{dz}^{(1)}\right] = 0, \tag{10}$$

$$\frac{\partial u_{dz}^{(1)}}{\partial \tau} - \upsilon_o \frac{\partial u_{dz}^{(2)}}{\partial \xi} + l_z u_{dz}^{(1)} \frac{\partial u_{dz}^{(1)}}{\partial \xi} - \nu l_z \frac{\partial \phi^{(2)}}{\partial \xi} = 0. \tag{11}$$

Now, using (7) – (11), one can eliminate $n_d^{(2)}$, $u_{dz}^{(2)}$, and $\phi^{(2)}$ and can obtain

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0. \tag{12}$$

This is the K-dV equation with the coefficients A and B given by

$$A = -\frac{\nu}{(1-\mu)^2}\left(\frac{1-\mu}{\nu}\right)^{3/2} \frac{l_z}{(1+\sigma\mu)^{3/2}}\left[1 + (3+\sigma\mu)\sigma\mu + \frac{1}{2}\mu(1+\sigma^2)\right],$$

$$B = \frac{1}{2}\left(\frac{1-\mu}{\nu}\right)^{3/2} \frac{l_z}{(1+\sigma\mu)^{3/2}}\left[1 + \frac{1-l_z^2}{\omega_{cd}^2}\right]. \tag{13}$$

The steady state solution of this K-dV equation is obtained by transforming the independent variables ξ and τ to $\eta = \xi - u_o\tau$ and $\tau = \tau$, where u_o is a constant velocity normalized to C_d, and imposing the appropriate boundary conditions, *viz.*, $\phi \to 0$, $\frac{d\phi^{(1)}}{d\eta} \to 0$, $\frac{d^2\phi^{(1)}}{d\eta^2} \to 0$ at $\eta \to \pm\infty$. Thus one can express the steady state solution of the K-dV equation as

$$\phi^{(1)} = \phi_m^{(1)} \operatorname{sech}^2 [(\xi - u_o \tau)/\delta], \tag{14}$$

where the amplitude $\phi_m^{(1)}$ and the width δ (normalized to λ_{Dd}) are given by

$$\phi_m^{(1)} = 3u_o / A,$$

$$\delta = \sqrt{4B/u_o}. \tag{15}$$

As $u_o > 0$, it is clear that finite amplitude solitary waves with $\phi > (<) 0$ exists if $A > (<) 0$. Therefore, for negatively charged dust grains ($\nu = 1$ and $\mu < 1$) $A < 0$, *i.e.*, there exists solitary waves with negative potential only, and for positively charged dust grains ($\nu = -1$ and $\mu > 1$) $A > 0$, *i.e.*, there exists solitary waves with positive potential only. It is obvious from (13) – (15) that the amplitude of these solitary waves decreases with the increase of l_z ($l_z = \cos\theta$ with θ being the angle between the directions of the wave propagation vector $\underline{k}$ and the external magnetic field $\underline{B}_o$. It is seen that the magnitude of the external magnetic field has no any effect on the amplitude of the solitary waves.

The effects of the magnitude of the external magnetic field ($\underline{B}_o$) and propagation angle (θ) on the width (δ) of these solitary waves are displayed in Figures 1 and 2. It is found from the Figure 1 that as we increase the magnitude of the magnetic field, the width of these solitary waves decreases, *i.e.*, external magnetic field makes the solitary structures more spiky. It is

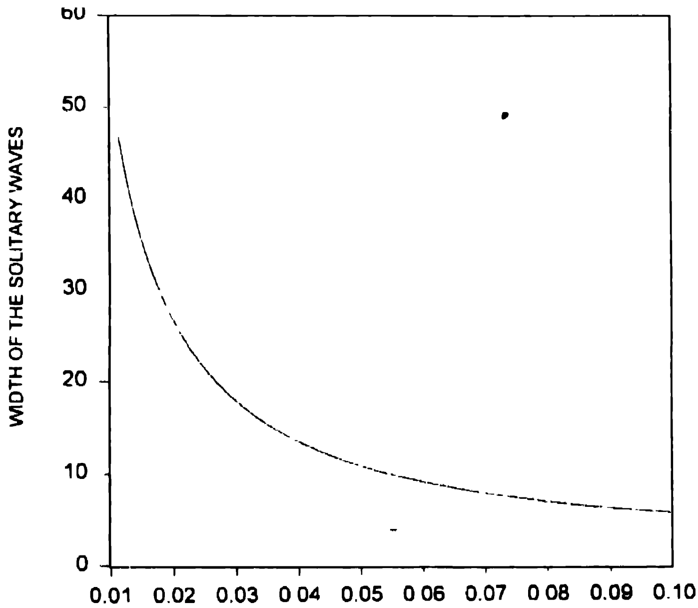

Figure 1. Width (δ) of the solitary waves is plotted against dust-cyclotron frequency (ω_{cd}) for $\sigma = 0.1$, $\mu = 0.1$, $u_o = 1.0$, and $l_z = 0.85$. Here the width δ is normalized to the Debye length λ_{Dd} and dust-cyclotron frequency ω_{cd} is normalized to dust-plasma frequency ω_{pd}

shown from the Figure 2 that as we increase the propagation angle (θ), the width (δ) increases for its lower range ($0^{\circ} < \theta < \sim 55^{\circ}$) but decreases for its higher range ($\sim 55^{\circ} < \theta < 90^{\circ}$). It is important to mention here that our analysis, which is only valid for small but finite amplitude limit, is not valid for large propagation angle (θ) that makes the amplitude large enough to break the assumptions ($\epsilon n_d^{(1)} < 1$, $\epsilon n_d^{(2)} < n_d^{(1)}$, *etc.*) used in the reductive perturbation method employed.

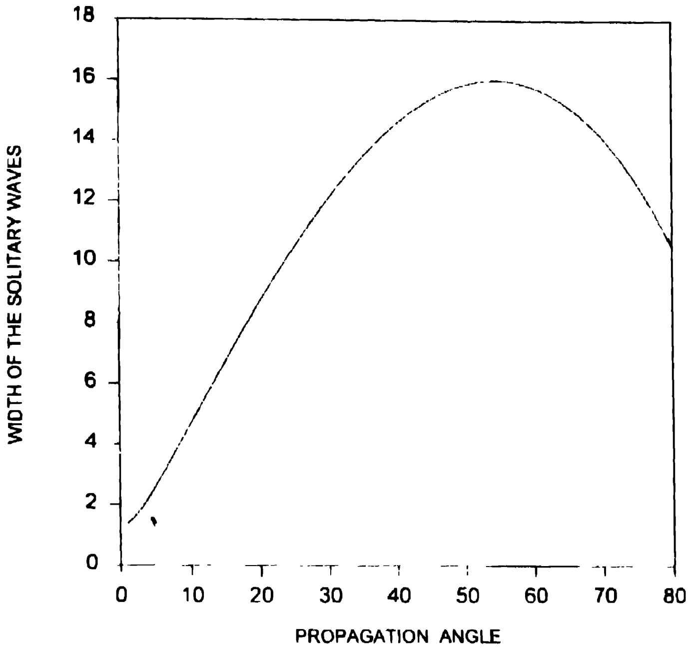

Figure 2. Width (δ) of the solitary waves is plotted against propagation angle (θ) for $\sigma = 0.1$, $\mu = 0.1$, $u_0 = 1.0$, and $\omega_{cd} = 0.05$. Here the width δ is normalized to the Debye length λ_{Dd}

It may be stressed here that the results of this investigation should be useful in understanding the nonlinear features of localized electrostatic disturbances in laboratory and space plasmas, in which negatively or positively charged dust particulates, and thermal electrons and ions are the plasma species.

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